January 2003: Begin with a Basic Formula for Sample Size (Rule 2.1)

Rules of the month are numbered in accordance with the numbering in the book. Thus, Rule 1.1 refers to the first rule in Chapter 1. And so on. These comments do not repeat the material in the book but highlights and amplifies it. A rule is stated as found in the book and then discussed.

Rule 2.1

"The basic formula is

$$n = \frac{16}{\Delta^2},$$

where,

$$\Delta = \frac{\mu_2 - \mu_1}{\sigma} = \frac{\delta}{\sigma}.$$

is the treatment difference to be detected in units of the standard deviation--the standardized difference. In the one-sample case the numerator is 8 instead of 16. This situation occurs when a single sample is compared with a known population value." (Note that I have changed the notation slightly.)

Further Comments on the Rule

At a recent site visit I attended a toxicologist was discussing a large number of animal studies he intended to carry out. One of the site visitors continued to press the toxicologist to say how many animals he would need. Finally, he was asked: “What kind of difference would you consider important.” He answered, “I am not very interested in less than two-fold differences.” This was as close as he could come to a specification of the alternative hypothesis. He did not give sample sizes but it occurred to me that a quick answer could be obtained as follows. The key issue is variability. I have argued in the book (page 37) that the variability in biological systems is often of the order of 35%. Assuming this is the case, I remembered that a coefficient of variation is approximately the standard deviation in the log scale (natural logarithms). And a two-fold difference in means translates to a difference in logs of ln(2)=0.6931 (carrying a few extra decimal places for intermediate calculations). Therefore given these conditions we can estimate the sample size to be approximately,

$$n = \frac{16(0.35)^2}{(\ln 2)^2} = 4.1 \approx 5.$$

So under this kind of scenario approximately 5 animals per group are needed. This depends, of course, on the variability being of the order 0f

35%. If the variability were greater, say 50%, then the sample sizes are on the order of 9 animals per group. Now if you look at the toxicological literature many studies use between 8 and 12 animals per group. These sample sizes are frequently used without statistical justification. But I wonder whether the toxicologists have instinctively settled on sample sizes that pick up reasonable treatment effects.

One could also argue as follows: A *k-fold* difference (assuming that *k>1*) can be translated as follows,

$$\frac{\mu_2}{\mu_1} = k \ .$$

This can be linked directly to the Proportionate Change discussed on page 35 of *Statistical Rules of Thumb* as follows,

$$\text{P.C.} = \frac{\mu_2 - \mu_1}{\mu_1} = \frac{\mu_2}{\mu_1} - 1 = k - 1 .$$

Thus the proportionate change formulation for sample size can be used. If this is done the sample sizes by both formulations turn out to be very similar.

I'm interested in collecting means and standard deviations for a diverse set of biological measurements to see whether the coefficient of variation argument above holds. If you have some of these kinds of data send them to me and I will collate them. What I would need are a brief description, the means and standard deviations, and a published source.